Cambridge International AS & A Level

Cambridge Assessment International Education
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/12**

Paper 1 **October/November 2017**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)

CWO Correct Working Only – often written by a ‘fortuitous’ answer

ISW Ignore Subsequent Working

SOI Seen or implied

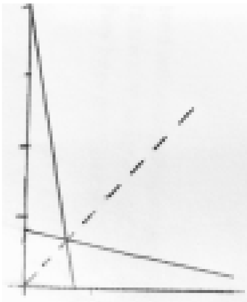
SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1	*EITHER:* Term is ${}^9C_3 \times 2^6 \times (-¼)^3$	**(B1, B1, B1)**	OE
	OR1: $\left(\frac{8x^3-1}{4x^2}\right)^9 = \left(\frac{1}{4x^2}\right)^9 \left(8x^3-1\right)^9$ *or* $-\left(\frac{1}{4x^2}\right)^9\left(1-8x^3\right)^9$		
	Term is $-\frac{1}{4^9} \times {}^9C_3 \times 8^6$	**(B1, B1, B1)**	OE
	OR2: $\left(2x\right)^9\left(1-\frac{1}{8x^3}\right)^9$		
	Term is $2^9 \times {}^9C_3 \times \left(-\frac{1}{8}\right)^3$	**(B1, B1, B1)**	OE
	Selected term, which must be independent of x $= -84$	**B1**	
		4	

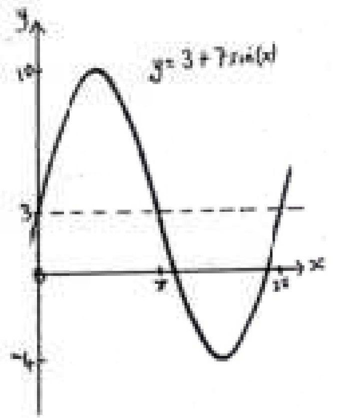
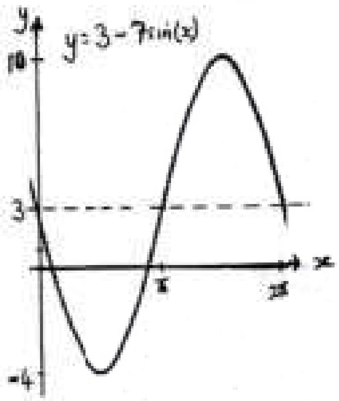
Question	Answer	Marks	Guidance
2(i)	$\frac{4-x}{5}$	B1	OE
	Equate a valid attempt at f^{-1} with f, or with x, or f with x $\rightarrow \left(\frac{2}{3}, \frac{2}{3}\right)$ **or** (0.667, 0.667)	M1, A1	Equating and an attempt to solve as far x =. Both coordinates.
		3	
2(ii)		B1	Line $y = 4 - 5x$ – must be straight, through approximately (0,4) and intersecting the positive x axis near (1,0) as shown.
		B1	Line $y = \frac{4-x}{5}$ – must be straight and through approximately (0, 0.8). No need to see intersection with x axis.
		B1	A line through (0,0) and the point of intersection of a pair of <u>straight</u> lines with negative gradients. This line must be at 45° unless scales are different in which case the line must be labelled $y=x$.
		3	

Question	Answer	Marks	Guidance
3(a)	Uses $r = (1.05 \text{ or } 105\%)^{9,\ 10 \text{ or } 11}$	B1	Used to multiply repeatedly or in any GP formula.
	New value = 10000×1.05^{10} = ($)16 300	B1	
		2	

Question	Answer	Marks	Guidance
3(b)	*EITHER:* $n = 1 \to 5 \qquad a = 5$	**(B1**	Uses $n = 1$ to find a
	$n = 2 \to 13$	**B1**	Correct S_n for any other value of n (e.g. $n = 2$)
	$a + (a + d) = 13 \quad \to d = 3$	**M1 A1)**	Correct method leading to $d =$
	OR: $\left(\frac{n}{2}\right)\left(2a+\left(n-1\right)d\right)=\left(\frac{n}{2}\right)\left(3n+7\right)$		$\left(\frac{n}{2}\right)$ maybe be ignored
	$\therefore dn + 2a - d = 3n + 7 \to dn = 3n \to d = 3$	**(*M1A1**	Method mark awarded for equating terms in n from correct S_n formula.
	$2a - \left(their\,3\right) = 7, \qquad a = 5$	**DM1 A1)**	
		4	

Question	Answer	Marks	Guidance
4(i)	Pythagoras → $r = \sqrt{72}$ OE or $\cos 45 = \frac{6}{r} \to r = \frac{6}{\cos 45} = 6\sqrt{2}$	**M1**	Correct method leading to r =
	Arc $DC = \sqrt{72} \times \frac{1}{4}\pi = \frac{3\sqrt{2}}{2}\pi$, 2.12π, 6.66	**M1 A1**	Use of $s=r\theta$ with their r (NOT 6) and $\frac{1}{4}\pi$
		3	
4(ii)	Area of sector-BDC is $\frac{1}{2} \times 72 \times \frac{1}{4}\pi$ (= 9π or 28.274…)	***M1**	Use of $\frac{1}{2}r^2\theta$ with their r (NOT 6) and $\frac{1}{4}\pi$
	Area $Q = 9\pi - 18$ (10.274…)	**DM1**	Subtracts their $\frac{1}{2} \times 6 \times 6$ from their $\frac{1}{2}r^2\theta$
	Area P is ($\frac{1}{4}\pi 6^2$ – area Q) = 18	**M1**	Uses {$\frac{1}{4}\pi 6^2$ – (their area Q using $\sqrt{72}$)}
	Ratio is $\frac{18}{9\pi - 18}$ $\left(\frac{18}{10.274}\right) \to 1.75$	**A1**	
		4	

Question	Answer	Marks	Guidance
5(i)	*EITHER:* Uses $\tan^2 2x = \dfrac{\sin^2 2x}{\cos^2 2x}$	(M1	Replaces $\tan^2 2x$ by $\dfrac{\sin^2 2x}{\cos^2 2x}$ not $\dfrac{\sin^2}{\cos^2}2x$
	Uses $\sin^2 2x = (1 - \cos^2 2x)$	M1	Replaces $\sin^2 2x$ by $(1 - \cos^2 2x)$
	$\rightarrow 2\cos^2 2x + 3\cos 2x + 1 = 0$	A1)	AG. All correct
	OR: $\tan^2 2x = \sec^2 2x - 1$	(M1	Replaces $\tan^2 2x$ by $\sec^2 2x - 1$
	$\sec^2 2x = \dfrac{1}{\cos^2 2x}$ Multiply through by $\cos^2 2x$ and rearrange	M1	Replaces $\sec^2 2x$ by $\dfrac{1}{\cos^2 2x}$
	$\rightarrow 2\cos^2 2x + 3\cos 2x + 1 = 0$	A1)	AG. All correct
		3	
5(ii)	$\cos 2x = -½, -1$	M1	Uses **(i)** to get values for $\cos 2x$. Allow incorrect sign(s).
	$2x = 120°, 240°$ or $2x = 180°1$ $x = 60°$ or $120°$	A1 A1 FT	**A1** for 60° or 120° FT for 180−1st answer
	or $x = 90°$	A1	Any extra answer(s) in given range only penalise fourth mark so max 3/4.
		4	

Question	Answer	Marks	Guidance
6(a)(i)	$4 = a + ½b$ $3 = a + b$	**M1**	Forming simultaneous equations and eliminating one of the variables – probably a. May still include $\sin\frac{\pi}{2}$ and / or $\sin\frac{\pi}{6}$
	$\to a = 5,\ b = -2$	**A1 A1**	
		3	
6(a)(ii)	$ff(x) = a + b\sin(a + b\sin x)$	**M1**	Valid method for ff. Could be f(0) = N followed by f(N) = M.
	$ff(0) = 5 – 2\sin 5 = 6.92$	**A1**	
6(b)	*EITHER:* $10 = c + d$ *and* $-4 = c – d$ $10 = c – d$ *and* $-4 = c + d$	**(M1**	Either pair of equations stated.
	$c = 3$, $d = 7$, -7 or ±7	**A1 A1)**	Either pair solved ISW **Alternately** c=3 **B1**, range = 14 **M1**→ d = 7, –7 or ±7 **A1**
	OR:  	**(M1 A1 A1)**	Either of these diagrams can be awarded M1.Correct values of c and/or d can be awarded the A1, A1
		3	

Question	Answer	Marks	Guidance
7(i)	$\frac{dy}{dx} = 2x - 4 = 0$		Can use completing the square.
	$\rightarrow x = 2,\ y = 3$	**B1 B1**	
	Midpoint of AB is (3, 5)	**B1 FT**	**FT** on (*their* 2, *their* 3) with (4,7)
	$\rightarrow m = \frac{7}{3}$ (or 2.33)	**B1**	
		4	
7(ii)	Simultaneous equations $\rightarrow\ x^2 - 4x - mx + 9(= 0)$	***M1**	Equates and sets to 0 must contain m
	Use of $b^2-4ac \rightarrow (m+4)^2 - 36$	**DM1**	Any use of b^2-4ac on equation set to 0 must contain m
	Solves = 0 → −10 or 2	**A1**	Correct end-points.
	$-10 < m < 2$	**A1**	Don't condone $\leqslant$ at either or both end(s). Accept $-10 < m$, $m < 2$.
		4	

Question	Answer	Marks	Guidance
8(i)	$\frac{dy}{dx} = 0$	**M1**	Sets $\frac{dy}{dx}$ to 0 and attempts to solve leading to two values for x.
	$x = 1, x = 4$	**A1**	Both values needed
		2	

Question	Answer	Marks	Guidance
8(ii)	$\frac{d^2y}{dx^2} = -2x + 5$	**B1**	
	Using both of their x values in their $\frac{d^2y}{dx^2}$	**M1**	Evidence of any valid method for both points.
	$x = 1 \rightarrow (3) \rightarrow$ Minimum, $x = 4 \rightarrow (-3) \rightarrow$ Maximum	**A1**	
		3	
8(iii)	$y = -\frac{x^3}{3} + \frac{5x^2}{2} - 4x \quad (+c)$	**B2, 1, 0**	$+c$ not needed. –1 each error or omission.
	Uses $x = 6$, $y = 2$ in an integrand to find c $\rightarrow c = 8$	**M1 A1**	Statement of the final equation not required.
		4	

Question	Answer	Marks	Guidance
9(i)	$\overrightarrow{AB} = \begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix}$ or $\overrightarrow{BA} = \begin{pmatrix} -4 \\ -3 \\ -2 \end{pmatrix}$	M1	Use of **b** – **a** or **a** – **b**
	e.g. $\overrightarrow{AO} . \overrightarrow{AB} = -8+6+2=0 \rightarrow O\hat{A}B = 90°$ AG **OR** $\lvert\overrightarrow{OA}\rvert = 3, \lvert\overrightarrow{OB}\rvert = \sqrt{38}, \lvert\overrightarrow{AB}\rvert = \sqrt{29}$ $OA^2 + AB^2 = OB^2 \rightarrow O\hat{A}B = 90°$ AG	M1 A1	Use of dot product with either $\overrightarrow{AO}$ *or* $\overrightarrow{OA}$ &*either* $\overrightarrow{AB}$ *or* $\overrightarrow{BA}$. Must see 3 component products OR Correct use of Pythagoras. In both methods must state angle or $\Theta = 90^{\circ}$ or similar for **A1**
		3	
9(ii)	$\overrightarrow{CB} = \begin{pmatrix} 6 \\ -6 \\ -3 \end{pmatrix}$ or $\overrightarrow{BC} = \begin{pmatrix} -6 \\ 6 \\ 3 \end{pmatrix}$	B1	Must correctly identify the vector.
	$\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC}$ (or $-\overrightarrow{CB}$) $= \begin{pmatrix} 0 \\ 7 \\ 4 \end{pmatrix}$	M1 A1	Correct link leading to $\overrightarrow{OC}$
		3	

Question	Answer	Marks	Guidance
9(iii)	$\lvert\overrightarrow{OA}\rvert = 3,\ \lvert\overrightarrow{BC}\rvert = 9,\ \lvert\overrightarrow{AB}\rvert = \sqrt{29}$ (5.39)	**B1**	For any one of these
	Area = ½(3 + 9) $\sqrt{29}$ or 3 $\sqrt{29}$ + 3 $\sqrt{29}$	**M1**	Correct formula(e) used for trapezium or (rectangle + triangle) or two triangles using their lengths.
	$= 6\sqrt{29}$ $(1\sqrt{1044}, 2\sqrt{261}$ ***or*** $3\sqrt{116})$	**A1**	Exact answer in correct form.
		3	

Question	Answer	Marks	Guidance
10(i)	$\dfrac{dy}{dx} = \dfrac{1}{2} \times (5x-1)^{-\frac{1}{2}} \times 5 \quad (= \dfrac{5}{6})$	**B1 B1**	**B1** Without × 5 **B1** × 5 of an attempt at differentiation
	m of normal = $-\dfrac{6}{5}$	**M1**	Uses $m_1 m_2 = -1$ with their numeric value from their dy/dx
	Equation of normal $\boldsymbol{y} - 3 = -\dfrac{6}{5}(\boldsymbol{x} - 2)$ OE or 5y + 6x = 27 or $\boldsymbol{y} = \dfrac{-6}{5}\boldsymbol{x} + \dfrac{27}{5}$	**A1**	Unsimplified. Can use $y = mx + c$ to get $c = 5.4$ ISW

Question	Answer	Marks	Guidance
10(ii)	*EITHER:*	**(B1**	Correct expression without ÷5
	For the curve $(\int)\sqrt{5x-1}\,dx = \dfrac{(5x-1)^{\frac{3}{2}}}{\frac{3}{2}} \div 5$	**B1**	For dividing an attempt at integration of y by 5
	Limits from $\frac{1}{5}$ to 2 used → 3.6 or $\frac{18}{5}$ OE	**M1 A1**	Using $\frac{1}{5}$ and 2 to evaluate an integrand $\left(\text{may be } \int y^2\right)$
	Normal crosses x-axis when $y = 0$, → x= (4½)	**M1**	Uses their equation of normal, NOT tangent
	Area of triangle = 3.75 or $\frac{15}{4}$ OE	**A1**	This can be obtained by integration
	Total area=3.6 + 3.75 = 7.35, $\frac{147}{20}$ OE	**A1)**	
	OR: For the curve: $(\int)\frac{1}{5}\left(y^2+1\right)dy = \frac{1}{5}\left(\frac{y^3}{3}+y\right)$	**(B2, 1, 0**	–1 each error or omission.
	Limits from 0 to 3 used → 2.4 or $\frac{12}{5}$ OE	**M1 A1**	Using 0 and 3 to evaluate an integrand
	Uses their equation of normal, NOT tangent.	**M1**	Either to find side length for trapezium or attempt at integrating between 0 and 3
	Area of trapezium = $\frac{1}{2}(2+4½)\times 3 = \frac{39}{4} \, or \, 9\frac{3}{4}$	**A1**	This can be obtained by integration
	Shaded area = $\frac{39}{4} - \frac{12}{5} = 7.35, \frac{147}{20}$ OE	**A1)**	

Question	Answer	Marks	Guidance
		7	